A simulation method to determine the minimum number of cracks in a Weibull's material, using a two model parameters

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Abstract

In this work a simulation method is employed to calculate the minimum number of defects that are currently found in a material whose behavior follows the Weibull's statistics, using a two model parameters m and σ_0. The strategy put forward is based upon the estimate and calculated values for these parameters m_s and σ_{0s} by means of a variable stress field, Evans' function and the division of the sample volume in N parts (equivalent to the number of cracks). In order to apply this simulation procedure, the experimental measurements of only m and σ_0 are required. As for the case of a glass assumed with a superficial brittleness, these parameters were obtained from the literature. The simulated results obtained for m_s and σ_{0s} were reasonable and fair, since the percentage difference when compared with the original ones was found to be fairly low. The plots of m_s and σ_{0s} versus N allow to find the minimum number of these defects (as we increase the number of interactions, we observe that the deviation is remarkable small). The numbers reported in this research work, when compared with other calculations for this material are to best of our understanding comparable in magnitude.

Keywords: Weibull theory; brittle failure; number of cracks

1. Introduction

The aim of fracture-statistics mechanics is to obtain the cumulative probability of the beginning of some failure in a solid body subjected to a known stress field. In the case of a constant uniaxial stress field, the fundamental formula establishing the connection between the cumulative probability of fragile fracture $F(\sigma)$ and the stress σ has been proposed by Weibull [1]

$$F(\sigma) = 1 - \exp\left\{-\frac{V}{V}\varphi(\sigma)\right\} \qquad (1)$$

where V_0 is the volume unit, V is the total volume of the body and $\varphi(\sigma)$ is the specific-risk function or Weibull's function. An expression that represents well the experimental results is

$$\varphi(\sigma) = \begin{cases} \left(\frac{\sigma-\sigma_L}{\sigma_0}\right)^m & \sigma\geq\sigma_L \\ 0 & 0\leq\sigma\leq\sigma_L \end{cases} \qquad (2)$$

in which σ_L is the inferior limit tension (below which there is no failure) and m and σ_0 are parameters that depend on the material and the manufacturing process. These are Weibull's parameters. It is necessary to notice, that the cumulative probability of failure increases when increasing the volume. The research on this subject has been concentrated fundamentally by obtaining the parameters that characterize the material. The evaluation of Weibull's parameters has been made by means of various methods. Between these, it is necessary to mention the analytical methods such as square minimums, maximum likelihood and chi-square [2]. Also graphical methods like nomograms have been used, for bending of rectangular section beams [3], compression of round section beams [4] and for the beams put under torsion [5]. The effect of Seewald-Karman has been considered as for the case of rectangular section beams [6,8], as for round ones [7,8]. The estimation if the parameters dispersion has carried out by means of the matrix of Fisher [9] for different states of stress [6, 7, 10]. The same method has been used to calculate the m parameter considering local probability [11].

The present work, on the contrary, supposes that these parameters are well known and presents a methodology based on a simulation of the tests to estimate the minimum number of active defects that there are in a beam of circular or rectangular section put under flexion. Simultaneously, by means of this simulation it is possible perhaps to discern if the defects are in the volume or on the surface.

2. Simulation

Considering $\sigma_L = 0$, the Weibull's function is left

$$\varphi(\sigma) = \left(\frac{\sigma}{\sigma_0}\right)^m \qquad (3)$$

and the cumulated probability of fracture, in the case of a constant stress field in this case is

$$F(\sigma) = 1 - exp\left\{-\frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m\right\} \tag{4}$$

The Evans' function$\xi(\sigma)$ defined by $\ln(1/(1-F)$, is

$$\xi(\sigma) = \frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m \tag{5}$$

If the stress field is variable, and it is given by

$$\sigma(r) = \sigma f(r) \leq \sigma \tag{6}$$

where σ is the maximum stress on the beam, r is the position vector and $f(r)$ us a function that describes the stress field, cumulated probability of failure in volume is [12],

$$F = 1 - exp\left\{-\frac{1}{V_0}\int_V \varphi\big(\sigma(r)\big)dV\right\} \tag{7}$$

In general, in this formula, the part of volume in compression is eliminated because the failure probability is very small with respect to the part of volume sub missed in traction.

Evans' function is

$$\xi(\sigma) = \frac{1}{V_0}\int_V \varphi\left(\sigma(r)\right)dV \tag{8}$$

or

$$\xi(\sigma) = \frac{1}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m \int_V f(r)dV \tag{9}$$

The cumulated probability on surface in the case of variable stress field case with field is written by [12],

$$F = 1 - exp\left\{-\frac{1}{S_0}\int_s \varphi\left(\sigma(r)\right)dS\right\} \tag{10}$$

and corresponding Evans´function by

$$\xi(\sigma) = \frac{1}{S_0}\int_S \varphi\left(\sigma(r)\right)dS \tag{11}$$

or

$$\xi(\sigma) = \frac{1}{S_0}\left(\frac{\sigma}{\sigma_0}\right)^m \int_S f(\quad)dS \tag{12}$$

Where S_0 is the surface unit.

In the above mentioned the specific-risk function $\varphi(\sigma)$ may be a volumetric or superficial function, if the involved integral extends on the volume or the surface.

In all the cases, writing the natural logarithm of the Evans' function, one obtains the following expression,

$$\ln \xi\,(\sigma) = m \cdot \ln \sigma + C \tag{13}$$

where C depends on σ_0 , among other parameters. Equation (13) is a straight line when plotting $\ln \xi$ vs. $\ln \sigma$.

Measured m and σ_0 , the simulation proposed in this work can be detached in the following steps:

Taje a set of N_0 random numbers $\lambda_i{}^{(1)}$, with $0 \leq \lambda_i{}^{(1)} \leq 1$, also placed in increasing order. When a test is made and one obtains a failure stress, the cumulative probability of failure is $0 \leq F(\sigma_i) \leq 1$. As this probability has a random character, the N_0 numbers λ_i corresponds to the $F(\sigma_i)$ values, since all these values have equal probability according to the formula (13). In this way it is possible to be established that

$$0 \leq \lambda_i{}^{(1)} \equiv F(\sigma_i) \leq 1 \tag{14}$$

Calculate the fracture stress $\sigma_i{}^r$ with relation (4). It is obtained:

$$\sigma_i{}^r = \sigma_0 \left(\frac{V_0}{V} \ln\left[\frac{1}{1-\lambda_i{}^{(1)}}\right]\right)^{\frac{1}{m}} \tag{15}$$

In this expression, the values of σ_0 and m constitute data collected experimentally

In the case of a uniform stress field, the collection $\{\sigma_i{}^r\}$ (column vector) constitutes the simulation because from these values and relationship (13) σ_0 and m can be recuperated with a little dispersion

In the case of a uniform stress field and considerating N random series $\{\sigma_{ij}{}^r\}(i = 1,2, \dots N_0; j = 1,2, \dots N)$ and choosing the least value of each row, like it is shown in Table 1, one obtains a situation equivalent to amplify the volume N times in equation (15). For this fact, the volume is not modified if one considers a volume V/N in (15), makes N random series $\{\sigma_{ij}{}^r\}(i = 1,2, \dots\ N_0; j = 1,2, \dots N)$ and one chooses the least value of each level or row. In this case the fracture occurs in the point where $\sigma_{ij}{}^r$ takes the minimum value.

Table 1
N random series

Series 1	Series 2	....	Series N	Least value of each row
${\sigma^r}_{11}$	${\sigma^r}_{12}$	....	${\sigma^r}_{1N}$	${\sigma^r}_1$ min
${\sigma^r}_{21}$	${\sigma^r}_{22}$	....	${\sigma^r}_{2N}$	${\sigma^r}_2$ min
....	....	....	....	
....	....	....	....	
${\sigma^r}_{No1}$	${\sigma^r}_{N02}$	....	${\sigma^r}_{NoN}$	${\sigma^r}_{No}$ min

* In the case of a variable stress field given by relation (6), the volume in (15) is divided in N parts and one make N random series $\{{\sigma_{ij}}^r\}(i = 1,2, \ldots N_0; j = 1,2, \ldots N)$ and one chooses the least value of each level or row. These N_0 minimum values constitute the simulation. With this, the volume remains constant and one obtains N points where potentially there are cracks. So, N represents the number of defects in the volume V. The location of ${\sigma_i}^r$ in the sample is made by means of a set of random numbers $0 \leq \lambda^{(2)}, \lambda^{(3)}, \lambda^{(4)}, \ldots \leq 1$. The failure occurs in the first point reaching the fracture stress when σ (maximum stress in relationship 6) increases from zero value.

* Consider N_0 value (number of essays) of cumulative probability given by [12].

$$F_i = \frac{i-\frac{1}{2}}{N_0} \text{ with } i = 1,2,3m \ldots N_0. \tag{16}$$

* Notice that the N_0 values of F are placed in increasing order

* Plot $\ln(\ln(1/(1-F_i)$ versus $\ln(\sigma_i)$, where F_i is obtained from (16) and σ_i is the maximum value of $\sigma(x, y, z)$ in (6), and calculate, by means of the least square method, the equation of the line representing the obtained points. In other words, the expression (13) is represented. One obtains therefore a value of m_S simulated (slope of the straight line) and a value of σ_{0S} simulated (from the value of C). Figure 1 shows a simulation.

Repeat this process n times to obtain a mean value m_S and σ_{0S} and the respective standard deviation.

It notes that in this method the sample volume remains constant and the position of the defects and the rupture tension are determined (at most) by four random numbers.

In general if the stress field is variable the means values of m_S and σ_{0S} (simulated) must agree with the experimentally measured values of m and σ_0 from a certain value of $N = N_S$. This number represents the minimum number of defects that can have in the material.

This simulation was applied to a circular section beam of radio R and length L and to a rectangular section beam of wide b. height h and of length L, under different conditions of load.

3. Beam with circular section in traction with volumetric brittleness. A cylinder under axial load $\boldsymbol{P}$ with volumetric brittleness is considered. In this case the stress field is constant. It is written

$$\sigma(x, y, z) = \frac{P}{\pi R^2} \tag{17}$$

With this, Evans' function is

$$\xi(\sigma) = \ln\left(\frac{1}{1-F}\right) = \frac{\pi L\, R^2}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m \tag{18}$$

Taking the logarithm

$$\ln \xi\,(\sigma) = m_S \cdot \ln \sigma + C \tag{19}$$

where

$$C = \ln\left(\frac{\pi\, L\, R^2}{V_0 {\sigma_0}^{m_S}}\right) \tag{20}$$

and therefore

$$\sigma_{0S} = \left(\frac{\pi\, L\, R^2}{V_0 e^C}\right)^{\frac{1}{m_S}} \tag{21}$$

4. Beam with circular section in flexure with volumetric brittleness. A cylinder under transversal load $\boldsymbol{P}$ with volumetric brittleness is considered.

The stress field is,

$$0 \le \sigma(x, y, z) = \frac{2xy}{LR}\sigma \le \sigma = \frac{PL}{\pi R^3} \tag{22}$$

with

$$0 \le x \le L\,/\,2$$

$$0 \le z \le R \tag{23}$$

$$0 \le y \le R$$

Considering $y = r \sin \varphi$, the Evans function is written

$$\xi(\sigma) = \frac{4}{V_0}\int_{x=0}^{\frac{L}{2}} \int_{\varphi=0}^{\frac{\pi}{2}} \int_{r=0}^{R} \left(\frac{2xr\sin\varphi}{LR}\frac{\sigma}{\sigma_0}\right)^m dxrd\varphi dr \tag{24}$$

Integrating

$$\xi(\sigma) = \frac{LR^2\sqrt{\pi}}{V_0(m+1)(m+2)} \frac{\Gamma\left(\frac{m+1}{2}\right)}{\Gamma\left(\frac{m+2}{2}\right)} \left(\frac{\sigma}{\sigma_0}\right)^m \tag{25}$$

The expression (13) takes the form

$$\ln \xi(\sigma) = m_S \cdot \ln \sigma + \ln \left(\frac{LR^2\sqrt{\pi}}{V_0(m_S+1)(m_S+2)\cdot\sigma_0{}^{m_3}} \frac{\Gamma\left(\frac{\mathrm{m}_S+1}{2}\right)}{\Gamma\left(\frac{\mathrm{m}_S+2}{2}\right)} \right) \tag{26}$$

or

$$\ln \xi(\sigma) = m_S \cdot \ln \sigma + C$$

By a random way $\sigma_i(x, y, z)$ is written

$$\sigma_i(x, y, z) = \lambda_i{}^{(2)} \lambda_i{}^{(3)} \sin \lambda_i{}^{(4)} \sigma \tag{27}$$

where

$$\begin{aligned} \lambda_i{}^{(2)} &= \frac{2x_i}{L} \\ \lambda_i{}^{(3)} &= \frac{r_i}{R} \\ \lambda_i{}^{(4)} &= \frac{2\varphi_i}{\pi} \end{aligned} \tag{28}$$

with $0 \leq \varphi_i \leq \pi / 2$

Dividing the volume in N parts, the formula (15) allows the fracture stress,

$$\sigma_i{}^r = \sigma_0 \left(\frac{V_0 N}{\pi L R^2} \ln \left[\frac{1}{1-\lambda_i{}^{(1)}} \right] \right)^{\frac{1}{m}} \tag{29}$$

From expression (22) one obtains the stress

$$\sigma_i = \frac{\sigma_i{}^r}{\lambda_i{}^{(2)}\lambda_i{}^{(3)}\sin\lambda_i{}^{(4)}} \tag{30}$$

and from (26), the simulated value of σ_{0S}

$$\sigma_{0S} = \left(\frac{LR^2\sqrt{\pi}}{e^C(m_S+1)(m_S+2)\cdot\sigma_0{}^{m_S}}\frac{\Gamma\left(\frac{\mathrm{m_S}+1}{2}\right)}{\Gamma\left(\frac{\mathrm{m_S}+2}{2}\right)}\right)^{\frac{1}{m_S}} \tag{31}$$

In all flexure cases, the material fractures or breaks in the point where the smaller σ_i is obtained.

5. Beam with circular section in flexure with superficial brittleness. A cylinder under transversal load P in the middle point with superficial brittleness is considered.

The stress field is,

$$0 \le \sigma(x, y, z) = \frac{2xy}{LR}\sigma \le \sigma = \frac{PL}{\pi R^3} \tag{32}$$

with

$$0 \le x \le \frac{L}{2}$$
$$0 \le z = \sqrt{R^2 - y^2 \le R} \tag{33}$$
$$0 \le y \le R$$

Considering $y = R\sin\varphi$, the Evans functions results

$$\xi(\sigma) = \frac{4}{S_0}\int_{x=0}^{\frac{L}{2}}\int_{\varphi=0}^{\frac{\pi}{2}}\left(\frac{2xR\sin\varphi}{LR}\frac{\sigma}{\sigma_0}\right)^m dxRd\varphi \tag{34}$$

Integrating

$$\xi(\sigma) = \frac{LR\sqrt{\pi}}{S_0(m+1)}\frac{\Gamma\left(\frac{\mathrm{m_S}+1}{2}\right)}{\Gamma\left(\frac{m_S+2}{2}\right)}\left(\frac{\sigma}{\sigma_0}\right)^{m_3} \tag{35}$$

and taking the logarithm

$$\ln \xi\,(\sigma) = m_S \cdot \ln \sigma + \ln\left(\frac{LR\sqrt{\pi}}{S_0(m_S+1)\cdot{\sigma_0}^{m_S}}\frac{\Gamma\left(\frac{\mathrm{m}_{<s}+1}{2}\right)}{\Gamma\left(\frac{m_S+2}{2}\right)}\right) \qquad (36)$$

or

$$\ln \xi(\sigma) = m_S \cdot \ln \sigma + C$$

The stress $\sigma_i(x, y, z)$ is obtained randomly like

$$\sigma_i(x, y, z) = {\lambda_i}^{(2)} \sin {\lambda_i}^{(3)} \sigma \qquad (37)$$

with

$${\lambda_i}^{(2)} = \frac{2x_i}{L} \qquad (38)$$

$${\lambda_i}^{(3)} = \frac{2\varphi_i}{\pi}$$

The failure stress is

$${\sigma_i}^r = \sigma_0 \left(\frac{NS_0}{\pi LR} \ln\left[\frac{1}{1-{\lambda_i}^1}\right]\right)^{\frac{1}{m}} \qquad (39)$$

with which

$$\sigma_i = \frac{{\sigma_i}^r}{{\lambda_i}^{(2)} \sin {\lambda_i}^{(3)}} \qquad (40)$$

and for σ_{0S}

$$\sigma_{0S} = \left(\frac{LR\sqrt{\pi}\,\Gamma\left(\frac{m_S+1}{2}\right)}{S_{0e}{}^{C}(m_S+1)\Gamma\left(\frac{m_S+2}{2}\right)}\right)^{\frac{1}{m_S}} \qquad (41)$$

6. Beam with rectangular section in flexure with volumetric brittleness. A beam of rectangular section, wide b and height h, under transversal load P in the middle point with volumetric brittleness is considered.

In this case the stress field is,

$$0 \leq \sigma(x, y, z) = \frac{4xy}{Lh}\sigma \leq \sigma = \frac{3PL}{2bh^2} \tag{42}$$

where

$$0 \leq x \leq L/2$$
$$0 \leq y \leq h/2 \tag{43}$$
$$0 \leq z \leq b$$

Evans' function is

$$\xi(\sigma) = \frac{4}{V_0}\int_{x=0}^{\frac{L}{2}}\int_{y=0}^{\frac{h}{2}}\int_{z=0}^{\frac{b}{2}}\left(\frac{4xy}{Lh}\frac{\sigma}{\sigma_0}\right)^m dxdydz \tag{44}$$

from which one obtains

$$\xi(\sigma) = \frac{bhL}{2V_0(m+1)^2}\left(\frac{\sigma}{\sigma_0}\right)^m \tag{45}$$

and

$$\ln\xi(\sigma) = m_s \cdot \ln\sigma + \ln\left(\frac{bhL}{2V_0(m_s+1)^2 \cdot {\sigma_0}^{m_s}}\right) \tag{46}$$

or

$$\ln\xi(\sigma) = m_s \cdot \ln\sigma + C$$

The stress $\sigma_i(x, y, z)$ can be written like

$$\sigma_i(x, y, z) = {\lambda_i}^{(2)}{\lambda_i}^{(3)}\sigma \tag{47}$$

with

$${\lambda_i}^{(2)} = \frac{4x_i}{L}{\lambda_i}^{(3)} = \frac{y_i}{h} \tag{48}$$

Dividing the volume in N parts, it is obtained

$$\sigma_i{}^r = \sigma_0 \left(\frac{V_0 N}{bhL} \ln\left[\frac{1}{1-\lambda_i{}^{(1)}}\right]\right)^{\frac{1}{m}} \tag{49}$$

and the stress σ_i

$$\sigma_i = \frac{\sigma_i{}^r}{\lambda_i{}^2 \lambda_i{}^3} \tag{50}$$

The simulated value of σ_{0s} is

$$\sigma_{0s} = \left(\frac{bhL}{2V_{0e}{}^C (m_s+1)^2}\right)^{\frac{1}{m_s}} \tag{51}$$

7. Beam with rectangular section in flexure with superficial brittleness. A beam of rectangular section, wide b and height h, under transversal load P in the middle point with superficial brittleness is considered.

The stress field is,

$$0 \le \sigma(x,y,z) = \frac{4xy}{Lh}\sigma \le \sigma = \frac{3PL}{2bh^2} \tag{52}$$

where

$$0 \le x \le L/2$$

$$0 \le y \le h/2 \tag{53}$$

$$0 \le z \le b$$

At the inferior face

$$\sigma(x, h/2\,, z) = \frac{2x}{L}\sigma \tag{54}$$

and the lateral faces

$$\sigma(x, y, \pm b/2) = \frac{4xy}{Lh}\sigma \tag{55}$$

In this case Evans' function is

$$\xi(\sigma) = \frac{2}{S_0}\int_{x=0}^{\frac{L}{2}}\int_{z=0}^{b}\left(\frac{2x}{L}\frac{\sigma}{\sigma_0}\right)^m dxdz + \frac{4}{S_0}\int_{x=0}^{\frac{L}{2}}\int_{y=0}^{\frac{h}{2}}\left(\frac{4xy}{Lh}\frac{\sigma}{\sigma_0}\right)^m dxdy \tag{56}$$

Integrationg one obtains

$$\xi(\sigma) = \frac{L}{S_0(m+1)}\left(b + \frac{h}{m+1}\right)\left(\frac{\sigma}{\sigma_0}\right)^m \tag{57}$$

and taking the logarithm

$$\ln \xi(\sigma) = m_s \cdot \ln \sigma + \ln\left(\frac{L}{S_0(m_s+1)\cdot{\sigma_0}^{m_s}}\left(b + \frac{h}{m_s+1}\right)\right) \tag{58}$$

or

$$\ln \xi(\sigma) = m_s \cdot \ln \sigma + C$$

Randomly, the stress $\sigma_i(x, y, z)$ is written

$$\sigma_i(x, h/2, z) = {\lambda_i}^{(2)}\sigma \tag{59}$$

where

$${\lambda_i}^{(2)} = \frac{2x_i}{L} \tag{60}$$

and

$$\sigma(x, y, \pm b/2) = {\lambda_i}^{(3)}{\lambda_i}^{(4)}\sigma \tag{61}$$

with

$${\lambda_i}^{(3)} = \frac{2x_i}{L}{\lambda_i}^{(4)} = \frac{2y_i}{h} \tag{62}$$

The failure stress is

$${\sigma_i}^r = \sigma_0\left(\frac{NS_0}{L(b+2h)}\ln\left[\frac{1}{1-{\lambda_i}^{(1)}}\right]\right)^{\frac{1}{m}} \tag{63}$$

with which

$$\sigma_i = \frac{\sigma_i{}^r}{\lambda_i{}^{(2)}} \quad and \quad \sigma_i = \frac{\sigma_i{}^r}{\lambda_i{}^{(3)}\lambda_i{}^{(4)}} \tag{64}$$

and for σ_{0S}

$$\sigma_{0s} = \left(\frac{L}{S_{0e}{}^c}\left(b + \frac{h}{m_s+1}\right)\right)^{\frac{1}{m_s}} \tag{65}$$

8. Application to a real case

The exposed method was applied to a circular glass beam of section of radio 2 mm and 50 mm long ($V = 628.32\ mm^3; S = 628.32mm^2$) and to a square section beam of wide 3,5449 mm and 50 mm long ($V = 628.32\ mm^3$;S=708.98 mm^2). The experimental values of m and X, for the superficial brittleness, were measured by Diaz et al. [13] for a circular section beam at Material Laboratory of Chili University. These values are indicated in Table 2.

Table 2
Weibull's parameters for a glass beam circular

Parameter	
m	7.5 ± 0.6
σ_{0vol}	$6.79 \pm 2[MPa]$
σ_{0sup}	$21 \pm 4[MPa]$

The parameter σ_{0vol} for volumetric brittleness was calculated by formula

$$\sigma_{0vol} = \sigma_{0sup}\left(\frac{S_0}{V_0}\frac{R}{m+2}\right)^{\frac{1}{m}} \tag{66}$$

which was obtained equaling the superficial Evans' function with the volumetric one and conserving the same m. The same was done for the standard deviation calculation.

8.1 Cylinder in traction.

Figure 2 shows curves $\ln\xi\ vs. \ln\sigma$ for different values of N, where N is an integer positive number. Two cases may be distinguished:

a) N dividing the volume $V = \pi \cdot 200mm^3$. In this case, it is observed that the curves agree like it was pointed out above.

b) N amplifying the volume. In this case, the simulated straight lines move parallelly to themselves. For instance, the line for $N = 10$, is displaced in $\ln 10$ with respect to the straight line for $N = 1$, like it is predicted by the theory.

Figure 3 shows the curves $m_s\ vs.$ N, where N represents the number of defects. It is observed that the simulated values practically reach the real values from N = 1. This case demonstrates that the simulation method is reliable since it reproduces the theory exactly.

8.2 Beams of cylindrical and square section in flexion

Figure 4 shows a simulation of the m parameter for a circular section beam with volumetric brittleness. The curves represent the mean value and the standard deviation to each value of *N*. It is observed that from a certain value of *N*, the average value becomes stabilized and that the original value falls within the permissible values of the simulated parameter. This happens in all the cases.

In all these N_0 simulations and the number of repetitions *n* were $N_0 = 100\ and\ n = 50$.

Figures 5 to 8 show, as an example, the curves $m_s vs.$N and $\sigma_{0s}\ vs.\ N$ for the following simulations in bending: cylindrical section beam with volumetric brittleness and square secion beam with superficial brittleness. In all these cases the pointed line represents the average mean value (m_s, σ_{0s}) minus the standard deviation between *N = 700* and *N = 1400*. An estimation of the number of defects is obtained where pointed straight lines cut the curve.

Table 3 indicates the average values simulated of m_s and σ_{0s} and its standard deviation, for each case, calculated *N* between 700 and 1400. Also it appears a comparison with the measured values of Table 2.

Table 4 shows the minimum number of defects taken from the curves $m_s vs.\ N(Nm), \sigma_{0s} vs.\ m(N\sigma_0)$ the average value between both (*N*), and finally the values by unit of volume (*N/V*) and surface (*N/S*).

TABLE 3
Simulated values of X

Section	Brittleness	m_s	σ_{0s} $[MPa]$	$(m - m_s)/m$ %	$(\sigma_0 - \sigma_{0s})/\sigma_0$ %
Circular	Volume	7.34 ± 0.76	6.56 ± 1.62	2.5	3.4
	Surface	7.53 ± 0.76	19.21 ± 2.70	0.4	8.5
Square	Volume	7.39 ± 0.76	6.03 ± 1.82	1.5	11.3
	Surface	7.40 ± 0.75	23.10 ± 3.13	1.3	10

TABLE 4
Minimum number of defects

Section	Brittleness	*Nm*	$N\sigma_0$	*N*	N/V $1/mm^3$	N/S $1/mm^2$
Circular	Volume	220	250	235	0.75	-------
	Surface	170	160	165	------	0.47
Square	Volume	160	170	165	0.53	------
	Surface	170	150	160	------	0.45

9. Conclusions

The proposed method assumes the presence of small defects in the sample volume and allows calculate the minimum number of defects present in the material so it can be analyzed with Weibull's statistics with two parameters.

The simulation reproduces acceptably the original values of *m* and σ_0, since, as is observed in table 2, the percentage differences are very low. By means of the proposed method, a theorical relation is established between m and minimum value of N. For *m = 7.5* is obtained theoretically $N \geq 75$[12]. With which the results obtained by simulation are in agreement with the theorical ones. Also, there is coincidence between the simulation and the theory in the fact that *m* does not depend neither on the sample shape nor on the location of the defects, that is to say, if they are in the volume or the surface of bodies. But, it is not possible to deduce by this way if the defects are in the volume or on the surface in this case. Nevertheless, there is a tendency that a concentration on the surface is closer to reality. Finally, the shape of bodies does not have much influence in the superficial concentration that favors it.

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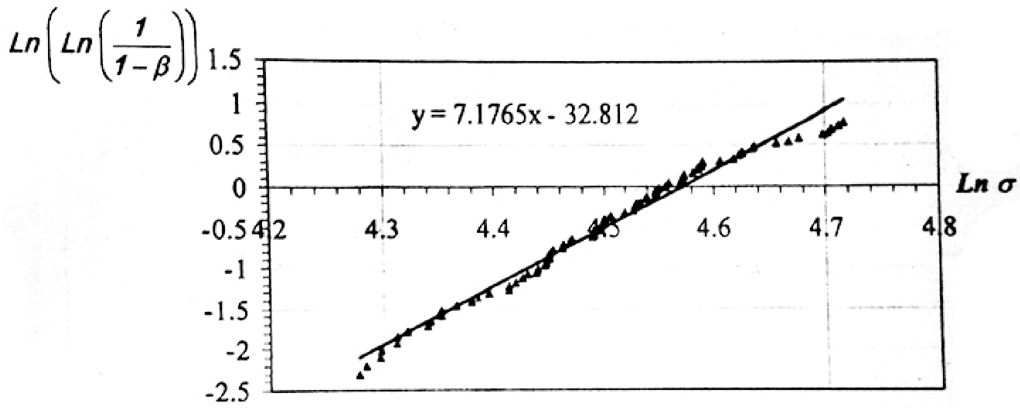

Figure 1. Representation of a simulation

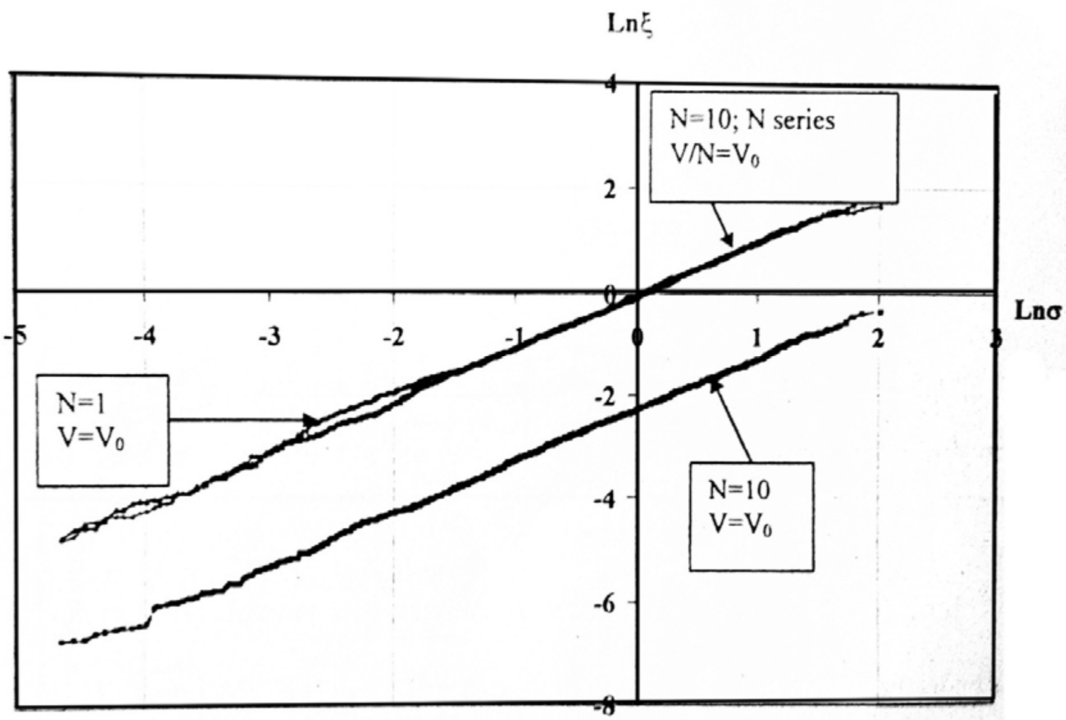

Figure 2. Cylinder in tension

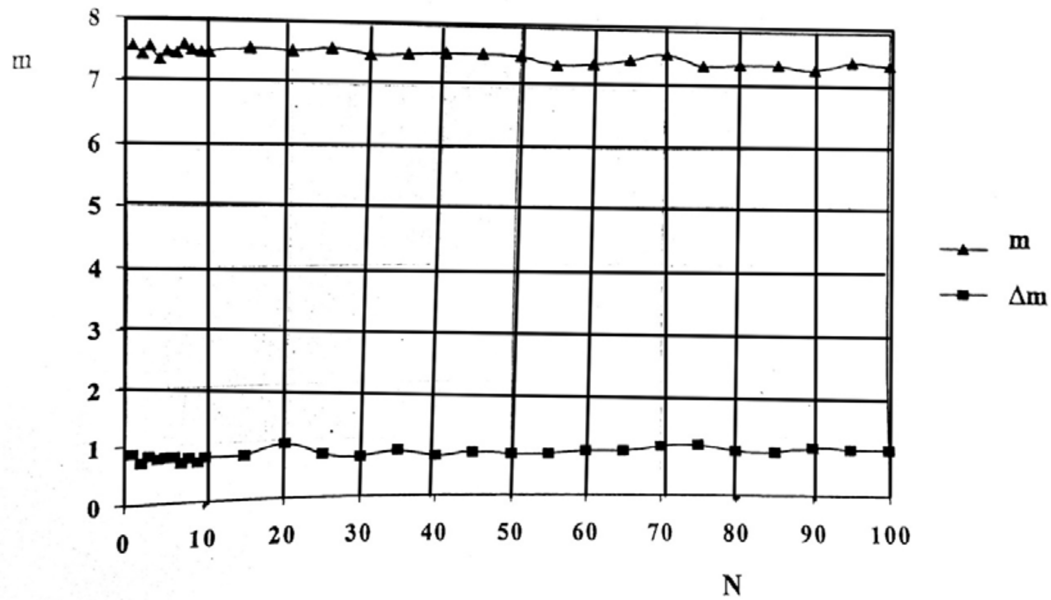

Figure 3. Cylinder in tension. ***m*** Simulated

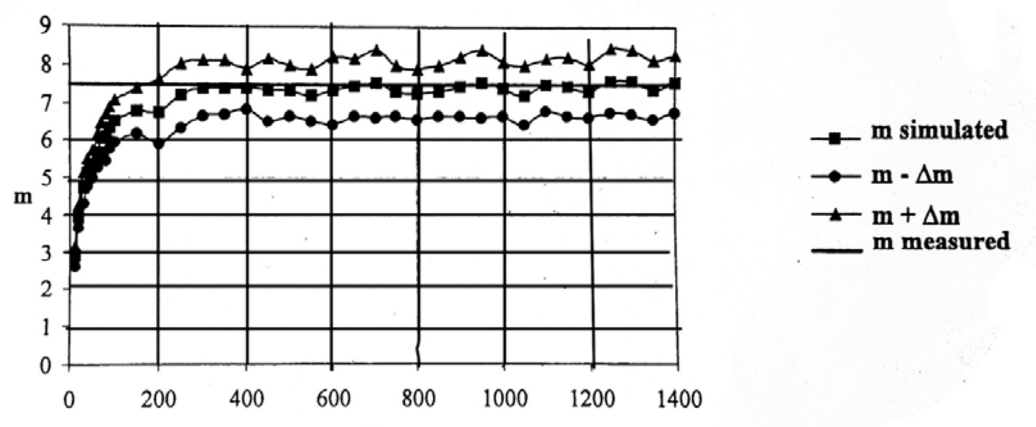

Figure 4. Simulation of parameter m of a circular section beam submitted to a transversal load in the middle point with volumetric brittleness.Δm Is the standard deviation

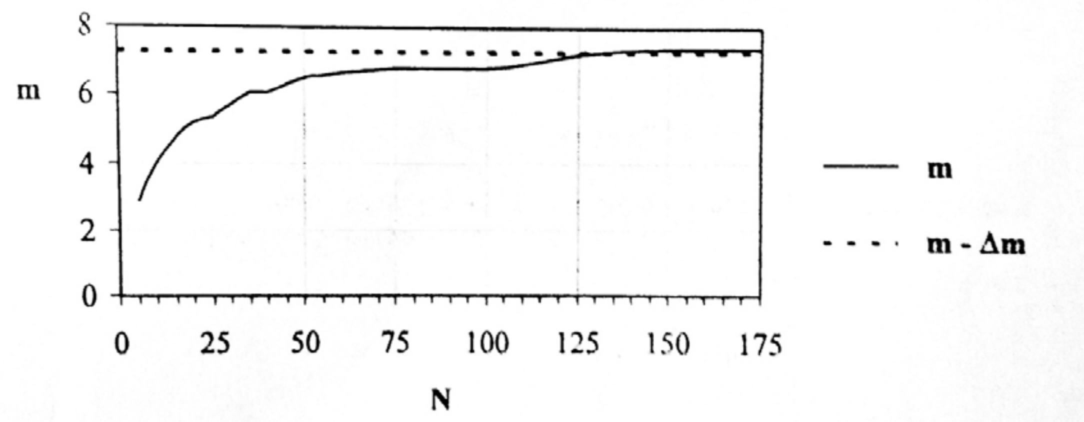

Figure 5. Circular section beam submitted to a transversal load in the middle point with volumetric brittleness. Parameter m

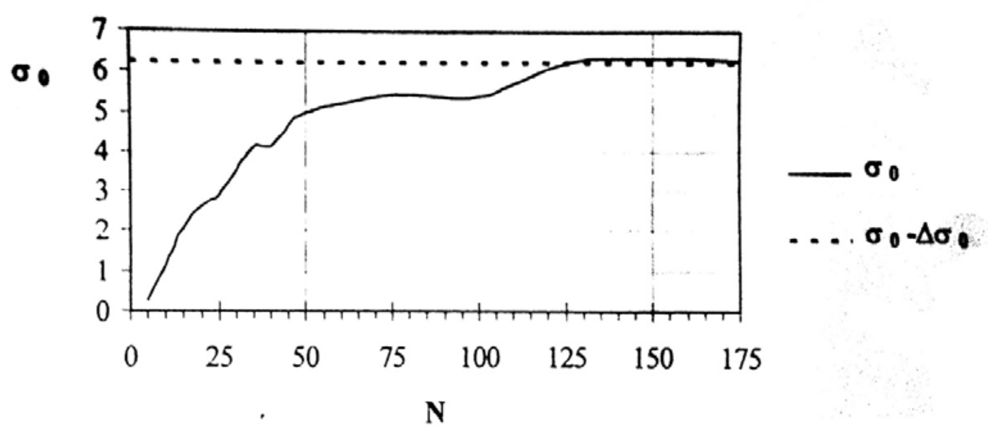

Figure 6. Circular section beam submitted to a transversal liad in the middle point with volumetric brittleness. Parameter σ_0

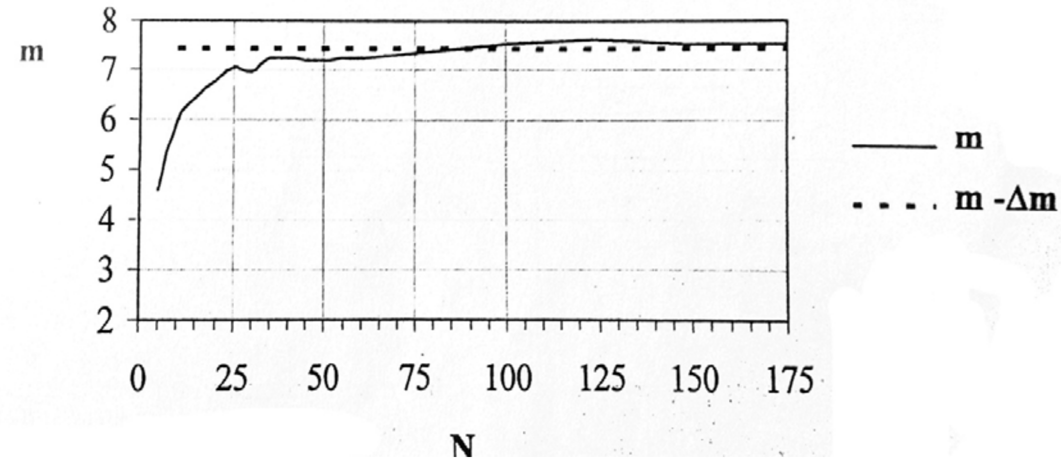

Figure 7. Square section beam submitted toa a transversal load in the middle point with superficial brittleness. Parameter m

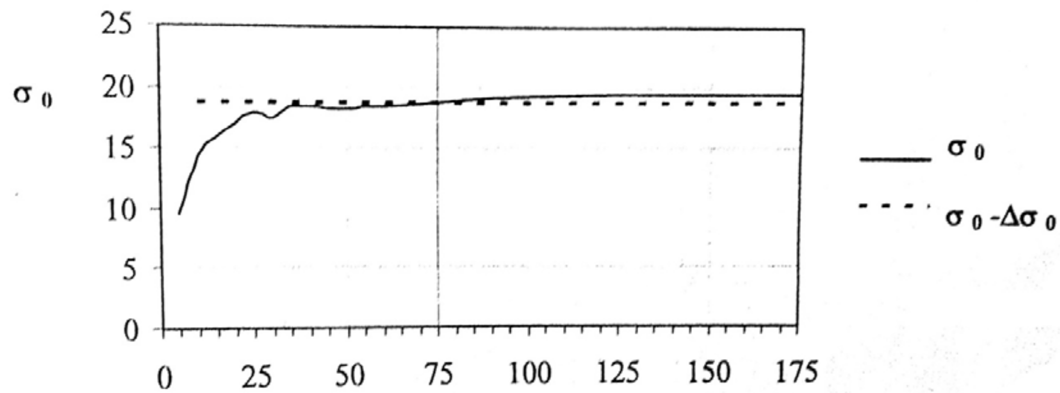

Figure 8. Square section beam submitted to a transversal load in the middle point with superficial brittleness. Parameter σ_0